



Project	PGSuper				Sheet	1
Subject	Spiral curve check calculations				of	1
Route	Made By	R. Brice		Supv	Date	12/9/2024

See reference attached at end of these calculations for referenced equations and figures.

### Compute Back and Forward Tangent Length

Back and forward tangent lengths are the same because the curve is symmetric.

Spiral Length,  $L_s = 145$  (Input from PGSuper)

Circular Curve Radius,  $R = 1063$  (Input from PGSuper)

$$\text{Spiral angle, } \Delta = \frac{L_s}{2R} = \frac{145}{(2)(1063)} = 0.0682 \text{ r (Eq. 12.15)}$$

$$X = L_s \left[ 1 - \frac{\Delta^2}{5(2!)} + \frac{\Delta^4}{9(4!)} \dots \right] = 145 \left( 1 - \frac{0.0682^2}{10} + \frac{0.0682^4}{216} \right) = 144.93 \text{ (Eq. 12.19)}$$

$$Y = L_s \left[ \frac{\Delta}{3} - \frac{\Delta^3}{7(3!)} + \frac{\Delta^5}{11(5!)} \dots \right] = 145 \left( \frac{0.0682}{3} - \frac{0.0682^3}{42} + \frac{0.0682^5}{1320} \right) = 3.29 \text{ (Eq. 12.20)}$$

$$X_o = X - R \sin \Delta = 144.93 - 1063 \sin 0.0682 = 72.48 \text{ (Eq. 12.11)}$$

$$o = Y - R(1 - \cos \Delta) = 3.29 - 1063(1 - \cos 0.0682) = 0.82 \text{ (Eq. 12.12)}$$

Total curve angle,  $I = 22^\circ 11' 49.66'' = 22.197^\circ = 0.3874 \text{ r (Input from PGSuper)}$

$$T_s = (R + o) \tan \frac{I}{2} + X_o = (1063 + 0.82) \tan \left( \frac{0.3874}{2} \right) + 72.48 = 281.16, \text{ (Eq. 12.13)}$$

PGSuper computes Back and Forward Tangent Length to be 281.175 ft.

### Compute TS Station

PI Station = 2356+17.96 (Input from PGSuper)

$$\text{TS Station} = \text{PI Station} - T_s = 2356+17.96 - 281.16 = 2353+36.78$$

PGSuper reports TS Station = 2353+36.78

### Compute ST Station

$$\text{Circular curve angle, } I_c = I - 2\Delta = 0.3874 - 2(0.0682) = 0.2510 \text{ (See Fig 12.10)}$$

$$\text{Circular curve arc length, } L_c = R I_c = 1063(0.2510) = 266.82$$

$$\text{Total curve length, } L = 2L_s + L_c = 2(145) + 266.82 = 556.82$$

$$\text{ST Station} = \text{TS Station} + \text{Total curve length} = 2353+36.78 + 556.82 = 2358+93.60$$

PGSuper reports ST Station = 2358+93.60

SCHMIDT  
& WONG

THIRD EDITION

# FUNDAMENTALS OF SURVEYING

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PWS  
Engineering

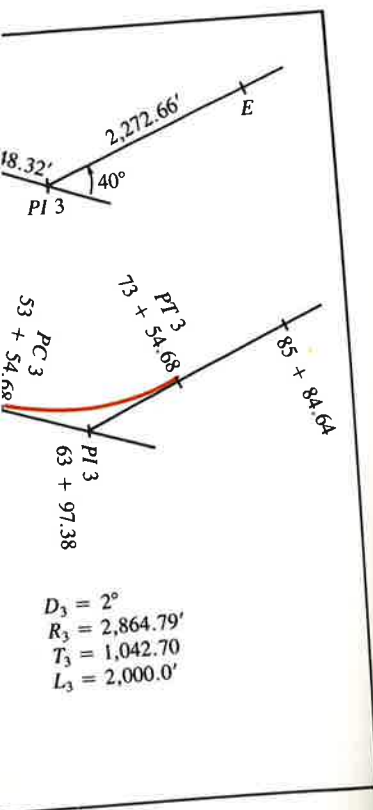
FUNDAMENTALS  
OF SURVEYING

## 12.12

## EQUAL-TANGENT SPIRALED CIRCULAR CURVE

The geometry of an equal-tangent spiraled circular curve is illustrated in Figure 12.10. It consists of an approach spiral of length  $L_s$ , a circular curve, and a leaving spiral also of length  $L_s$ . The point where the spiral departs from the tangent is called  $TS$  (tangent to spiral). At  $TS$ , the curvature of the spiral is  $0^\circ$ . The curvature gradually increases along the spiral until it reaches the degree of curvature of the circular curve at point  $SC$  (spiral to curve). The point at which the circular curve runs into the spiral is denoted as  $CS$  (curve to spiral); and the leaving spiral meets the forward tangent at point  $ST$  (spiral to tangent).

Point  $O$  in Figure 12.10 is the center of the circular curve. The line  $OB$  is perpendicular to the back tangent and is parallel to the line joining  $O'$  and  $TS$ . When the circular curve is extended beyond  $SC$ , it meets line  $BO$  at point  $G$ . The distance  $BG$  is called the *throw* of the spiral and is usually denoted by the letter  $o$ . The distance along the back tangent between points  $B$  and  $TS$  is denoted as  $X_o$ . The position of point  $SC$  with respect to the point  $TS$  is defined by the distances  $X$  along the tangent and the offset distance  $Y$ . The angle,  $\Delta$ , subtended by the line  $BO$  and the radius at  $SC$  is called the spiral angle.



the center line of the route are  
ugh the  $PI$ s. If the arc definition of  
ioning provides the actual center  
the project.

## VES

brupt change in curvature occurs at  
of a 5-degree curve, the curvature is  
-speed routes, an easement curve is  
n from  $0^\circ$  to the degree of curvature  
provides a gradual transition from a  
perelevated cross-section along the  
nterbalance the effect of centrifugal  
st commonly used easement curves is  
characteristic that its curvature changes

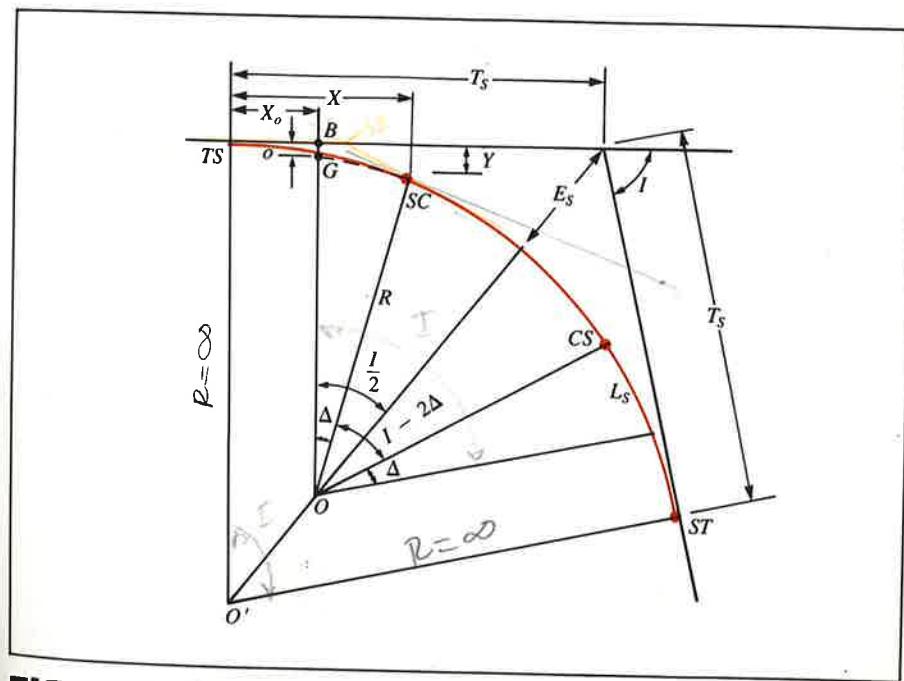


FIGURE 12.10 Equal-tangent spiraled circular curve



Let  $T_s$  denote the tangent distance between  $PI$  and  $TS$ ,  $E_s$  denote the external distance,  $I$  denote the intersection angle, and  $R$  denote the radius of the circular curve. Then the following relationships can be derived from Figure 12.10:

$$X_o = X - R \sin \Delta \quad (12.11)$$

$$o = Y - R(1 - \cos \Delta) \quad (12.12)$$

$$T_s = (R + o) \tan \frac{I}{2} + X_o \quad (12.13)$$

$$E_s = (R + o) \left( \frac{1}{\cos \frac{I}{2}} - 1 \right) + o \quad (12.14)$$

A spiral is usually defined by its length  $L_s$ . The spiral angle,  $\Delta$ , for a spiral of length  $L_s$  and connecting to a circular curve of  $D_a$  degrees can be computed from the following expression:

$$\Delta \text{ (in degrees)} = \frac{L_s D_a}{200} \quad (12.15)$$

$\Delta = \frac{L_s}{2R} \text{ (rad)}$

The values of  $X$  and  $Y$  in Eqs. (12.11) and (12.12) can be computed using Eqs. (12.17) and (12.18) given in the next section.

### 12.13

#### EQUATIONS OF A HIGHWAY SPIRAL

Let  $x$  and  $y$  denote the tangent and offset distances respectively of a point  $P$  located on the spiral (see Figure 12.11). Suppose that  $P$  is located at a distance  $l$ , measured along the spiral, from  $TS$ . Furthermore, let  $\delta$  be the angle, measured in radians, subtended by the distance  $l$ . Then the following relationships exist:

$$x = l \left[ 1 - \frac{\delta^2}{5(2!)} + \frac{\delta^4}{9(4!)} - \frac{\delta^6}{13(6!)} + \dots \right] = l \sum_{n=0}^{\infty} (-1)^n \frac{\delta^{2n}}{(1+4n)(2n)!} \quad (12.16)$$

and

$$y = l \left[ \frac{\delta}{3} - \frac{\delta^3}{7(3!)} + \frac{\delta^5}{11(5!)} - \frac{\delta^7}{15(7!)} + \dots \right] \quad (12.17)$$

where

$$\delta = \left( \frac{l}{L_s} \right)^2 \Delta \quad (12.18)$$

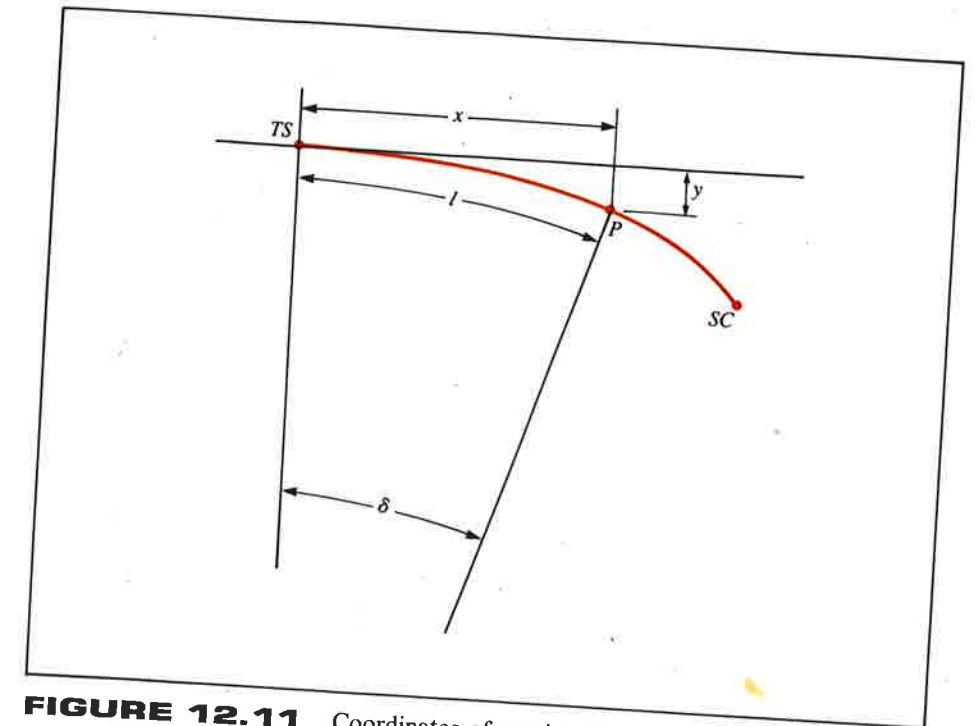


FIGURE 12.11 Coordinates of a point on a spiral

The preceding equations describe a spiral that is used universally in highway applications. A slightly different spiral is used in railroads.

Of particular interest is the tangent and offset distances ( $X$  and  $Y$ ) of point  $SC$ . Since  $l = L_s$  and  $\delta = \Delta$  at  $SC$ ,

$$X = L_s \left[ 1 - \frac{\Delta^2}{5(2!)} + \frac{\Delta^4}{9(4!)} - \frac{\Delta^6}{13(6!)} + \dots \right] \quad (12.19)$$

$$Y = L_s \left[ \frac{\Delta}{3} - \frac{\Delta^3}{7(3!)} + \frac{\Delta^5}{11(5!)} - \frac{\Delta^7}{15(7!)} + \dots \right] \quad (12.20)$$

Remember that  $\delta$  and  $\Delta$  are expressed in radians in Eqs. (12.16) to (12.20). For the derivations of Eqs. (12.15) to (12.20), readers are referred to References 12.1 and 12.2.

### 12.14

#### LAYOUT OF A SPIRAL

A spiral is usually located in the field using a number of equal sections. Figure 12.12 shows a spiral located in five equal sections. Point 1 on the spiral is located using the deflection angle  $a_1$  and a chord distance equal to  $L_s/5$ . Point 2 is then located using deflection angle  $a_2$  and a chord distance